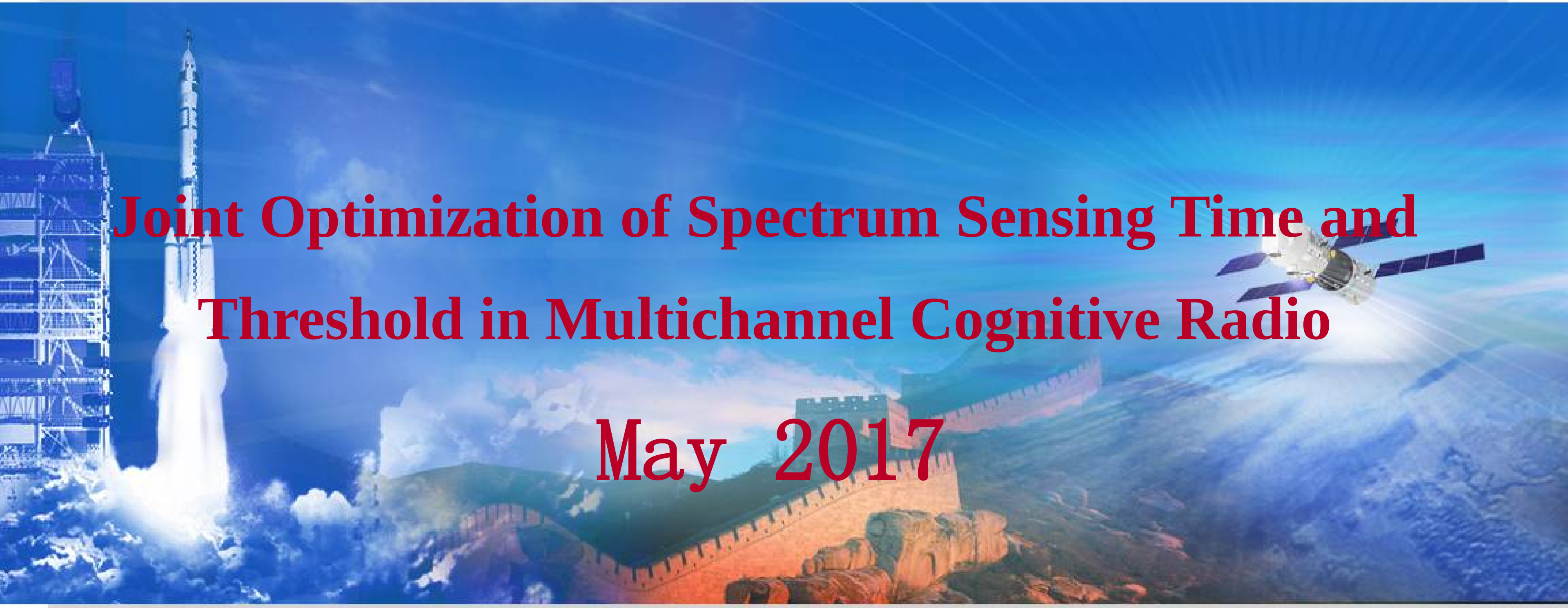

Joint Optimization of Spectrum Sensing Time and Threshold in Multichannel Cognitive Radio

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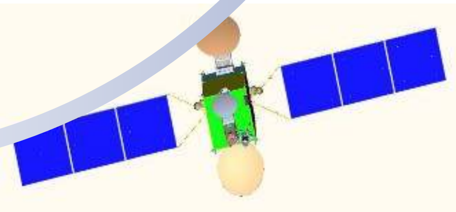
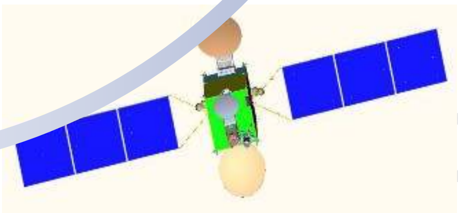
Agenda

- Introduction
- Energy Detection and System Model
- Optimization of Overall Throughput of SU
- Results and Discussion
- Conclusions

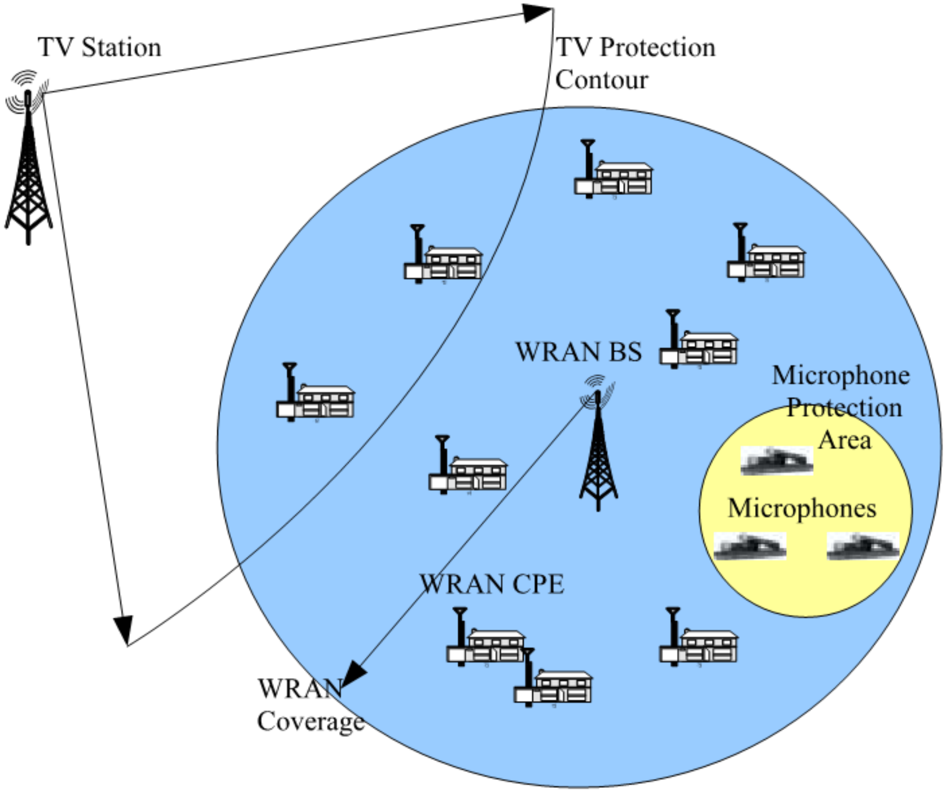


Introduction

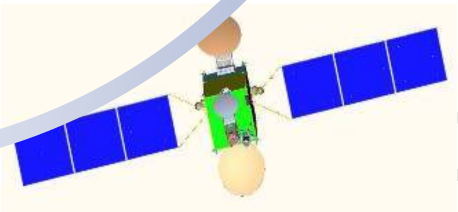


I Introduction

- **Cognitive Radio**
- **Throughput**
- **Sensing time**
- **Sensing threshold**
- **Joint Optimization**



Energy Detection and System Model



II. Energy Detection And System Model

The discrete received signal at the secondary user in sub channel l

$$y_l(t) = \begin{cases} n_l(t), H_l^0 \\ h_l s_l(t) + n_l(t), H_l^1 \end{cases}$$

$$t = (1, 2, 3, \dots M)$$

Test statistic for energy detector

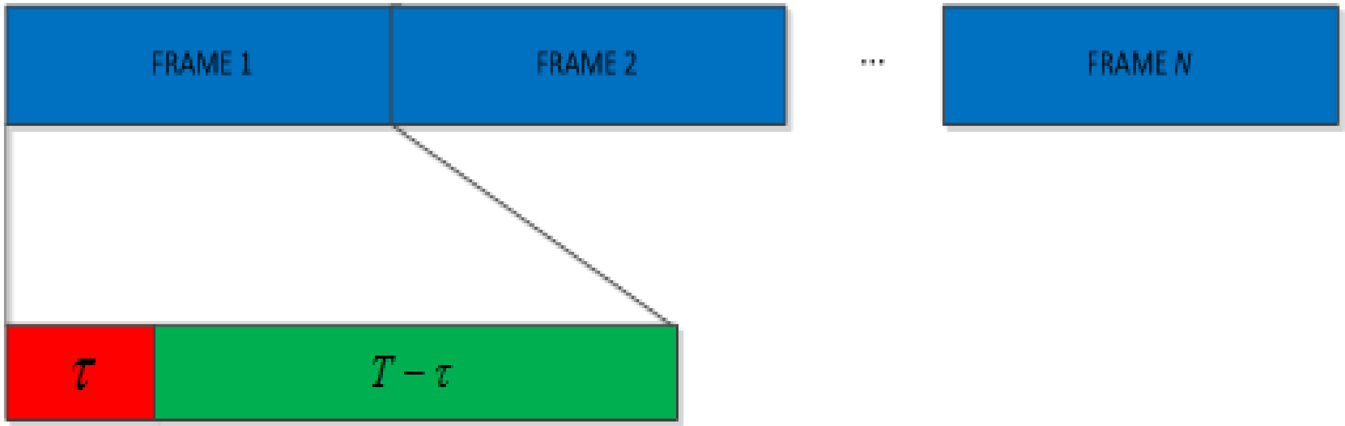
$$Y_l = \frac{1}{M} \sum_{t=1}^{M} \left| y_l(t) \right|^2$$

The probability of false alarm and the probability of detection are given by

$$\begin{cases} P_l^f(\tau, \varepsilon_l) = Q\left(\dfrac{\varepsilon_l - \sigma_l^2}{\sqrt{\sigma_l^4 / (\tau f_s)}} \right) \\ P_l^d(\tau, \varepsilon_l) = Q\left(\dfrac{\varepsilon_l - (1+\gamma_l)\sigma_l^2}{\sqrt{(1+2\gamma_l)\sigma_l^4 / (\tau f_s)}} \right) \end{cases}$$

II. Energy Detection And System Model

SU's listen-before-transmit frame structure



The throughput of the SU

$$c_l^0 = \log_2\left(1+\gamma_l^{SU}\right) \qquad c_l^1 = \log_2\left(1+\frac{\gamma_l^{SU}}{1+\gamma_l}\right)$$

The throughput of the PU

$$r_l^0 = \log_2\left(1+\gamma_l^{PU}\right) \qquad r_l^1 = \log_2\left(1+\frac{\gamma_l^{PU}}{1+\gamma^{PS}}\right)$$

II. Energy Detection And System Model

The aggregate throughput of the SU over all the sub channels

$$C(\tau,\varepsilon)=\left(\frac{T-\tau}{T}\right)\sum_{l=1}^{L}\Big(P\left(H_l^0\right)c_l^0\left(1-P_l^f\left(\tau,\varepsilon_l\right)\right)$$

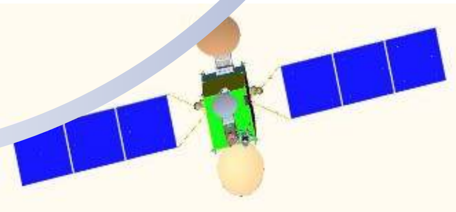
$$+P\left(H_l^1\right)c_l^1\left(1-P_l^d\left(\tau,\varepsilon_l\right)\right)\Big)$$

The aggregate throughput of the PU over all the sub channels

$$R(\tau,\varepsilon)=\frac{P\left(H_l^1\right)}{T}\tau\sum_{l=1}^{L}r_l^0$$

$$+(T-\tau)\sum_{l=1}^{L}\left(r_l^0P_l^d\left(\tau,\varepsilon_l\right)+r_l^1\left(1-P_l^d\left(\tau,\varepsilon_l\right)\right)\right)$$

Optimization of Overall Throughput of SU



III. Optimization Of Overall Throughput Of SU

The optimization model

$$\max_{\tau,\varepsilon} C(\tau,\varepsilon)$$

$$s.t. \begin{cases} R(\tau,\varepsilon) \geq \xi, \\ P_l^f(\tau,\varepsilon_l) \leq \alpha, \quad \alpha \leq 0.5 \\ P_l^d(\tau,\varepsilon_l) \geq \beta, l = 1,2,\cdots L. \quad \beta \geq 0.5 \end{cases}$$

This is a multiple variable optimization problem, and we use alternative optimization method to solve this problem.

III. Optimization Of Overall Throughput Of SU

Optimization of Sensing Threshold

$$\max_{\varepsilon} C(\varepsilon) = \left(\frac{T-\tau_0}{T}\right) \sum_{l=1}^{L} P\left(H_l^0\right) c_l^0 \left(1 - P_l^f\left(\varepsilon_l\right)\right)$$

$$+P\left(H_l^1\right) c_l^1 \left(1 - P_l^d\left(\varepsilon_l\right)\right)$$

$$s.t. \begin{cases} \sum_{l=1}^{L} \Delta r_l P_l^d\left(\varepsilon_l\right) \geq g\left(\tau_0\right) \\ \varepsilon_l^{\min} < \varepsilon_l < \varepsilon_l^{\max} \quad \left(l = 1, 2, \ldots L\right) \end{cases}$$

Where

$$\Delta r_l = r_l^0 - r_l^1$$

$$g\left(\tau_0\right) = \frac{\xi}{\eta_l^1\left(T-\tau_0\right)} - \frac{\tau_0}{T-\tau_0} \sum_{l=1}^{L} r_l^0 - \sum_{l=1}^{L} r_l^1$$

$$\varepsilon_{\min} = \left(\frac{Q^{-1}\left(\alpha\right)}{\sqrt{\tau_0 f_s}} + 1\right) \sigma_l^2$$

$$\varepsilon_{\max} = \left(Q^{-1}\left(\beta\right) \sqrt{\frac{2\gamma_l + 1}{\tau_0 f_s}} + \gamma_l + 1\right) \sigma_l^2$$

III. Optimization Of Overall Throughput Of SU

Optimization model of sensing threshold can be modified to unconstrained by integrating positive Lagrange multiplier, leading to

$$L(\varepsilon)=\sum_{l=1}^{L}\left(P\left(H_l^0\right)c_l^0\left(1-P_l^f\left(\varepsilon_l\right)\right)+P\left(H_l^1\right)c_l^1\left(1-P_l^d\left(\varepsilon_l\right)\right)\right)$$

$$+\lambda\left(\sum_{l=1}^{L}\Delta r_l P_l^d\left(\varepsilon_l\right)-g\left(\tau_0\right)\right)$$

Requiring the gradient of $\frac{\partial L(\varepsilon)}{\partial \varepsilon}=0$

$$\varepsilon_l^0=\left(\sqrt{\frac{1}{4}+\frac{1}{2}\gamma_l+\frac{\left(1+2\gamma_l\right)}{\left(\tau_0 f_s\right)\gamma_l}\ln\left(\frac{P\left(H_l^0\right)c_l^0\sqrt{\left(1+2\gamma_l\right)}}{\lambda\Delta r_l-P\left(H_l^1\right)c_l^1}\right)}\right.$$

$$\left.+\frac{1}{2}\right)\sigma_l^2$$

Optimal threshold:

$$\varepsilon_l^*=\begin{cases}\varepsilon_l^{\min}, \varepsilon_l^0<\varepsilon_l^{\min}\\ \varepsilon_l^0, \varepsilon_l^{\min}<\varepsilon_l^0<\varepsilon_l^{\max}\\ \varepsilon_l^{\max}, \varepsilon_l^0>\varepsilon_l^{\max}\end{cases}$$

$$\left(l=1,2,\ldots L\right)$$

III. Optimization Of Overall Throughput Of SU

Optimization of Sensing Time

$$P_l^f = Q\left(Q^{-1}\left(P_l^d\right)\sqrt{1+2\gamma_l} + \gamma_l\sqrt{\tau f_s}\right)$$

$$P_l^d\left(\tau,\varepsilon_l\right) = P_l^d\left(\tau_0,\varepsilon_l^*\right) = \mathrm{P}_l$$

$$\max_{\tau} C\left(\tau\right) = \left(\frac{T-\tau}{T}\right)\sum_{l=1}^{L} P\left(H_l^0\right)c_l^0(1-$$
$$Q\left(Q^{-1}\left(\mathrm{P}_l\right)\sqrt{1+2\gamma_l} + \gamma_l\sqrt{\tau f_s}\right)) + P\left(H_l^1\right)c_l^1\left(1-\mathrm{P}_l\right)$$
$$s.t. P_l^f\left(\tau,\varepsilon_l\right) \le \alpha, l = 1,2,\cdots L.$$

$$\tau \ge \max\left(\tau_1,\tau_2,\ldots\tau_L\right)$$

$$\nabla C\left(\tau_{\max}\right) = 0$$

$$\tau^* = \max\left(\tau_{\max},\max\left(\tau_1,\tau_2,\ldots\tau_L\right)\right)$$

III. Optimization Of Overall Throughput Of SU

Joint Optimization of Sensing Time and Sensing Threshold

1) Set the initial parameter: $k=1 \quad \tau^k=0, \varepsilon^k=\{0\} \quad \xi=0.01$

2) Set $\tau^k=\tau_0, \tau_0 \in [0,T]$

3) $$\varepsilon_l^0 = \left(\sqrt{\frac{1}{4}+\frac{1}{2}\gamma_l+\frac{(1+2\gamma_l)}{(\tau_0 f_s)\gamma_l}\ln\left(\frac{P(H_l^0)c_l^0\sqrt{(1+2\gamma_l)}}{\lambda\Delta r_l - P(H_l^1)c_l^1}\right)} + \frac{1}{2}\right)\sigma_l^2$$

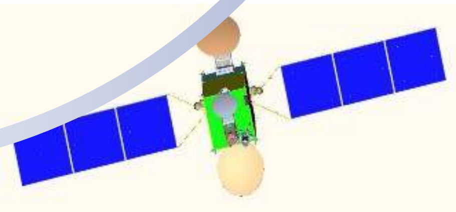
4) Set $\varepsilon^{k+1}=\varepsilon^*$

5) $\tau^*=\max(\tau_{\max}, \max(\tau_1, \tau_2, \ldots \tau_L))$

6) Set $\tau^{k+1}=\tau^*, k=k+1$

7) Repeat 3) ~ 6) until $\left|\tau^k-\tau^{k-1}\right| \le \xi, \left|\varepsilon^k-\varepsilon^{k-1}\right| \le \xi$

Results and Discussion



IV. Results And Discussion

The simulation parameter is :

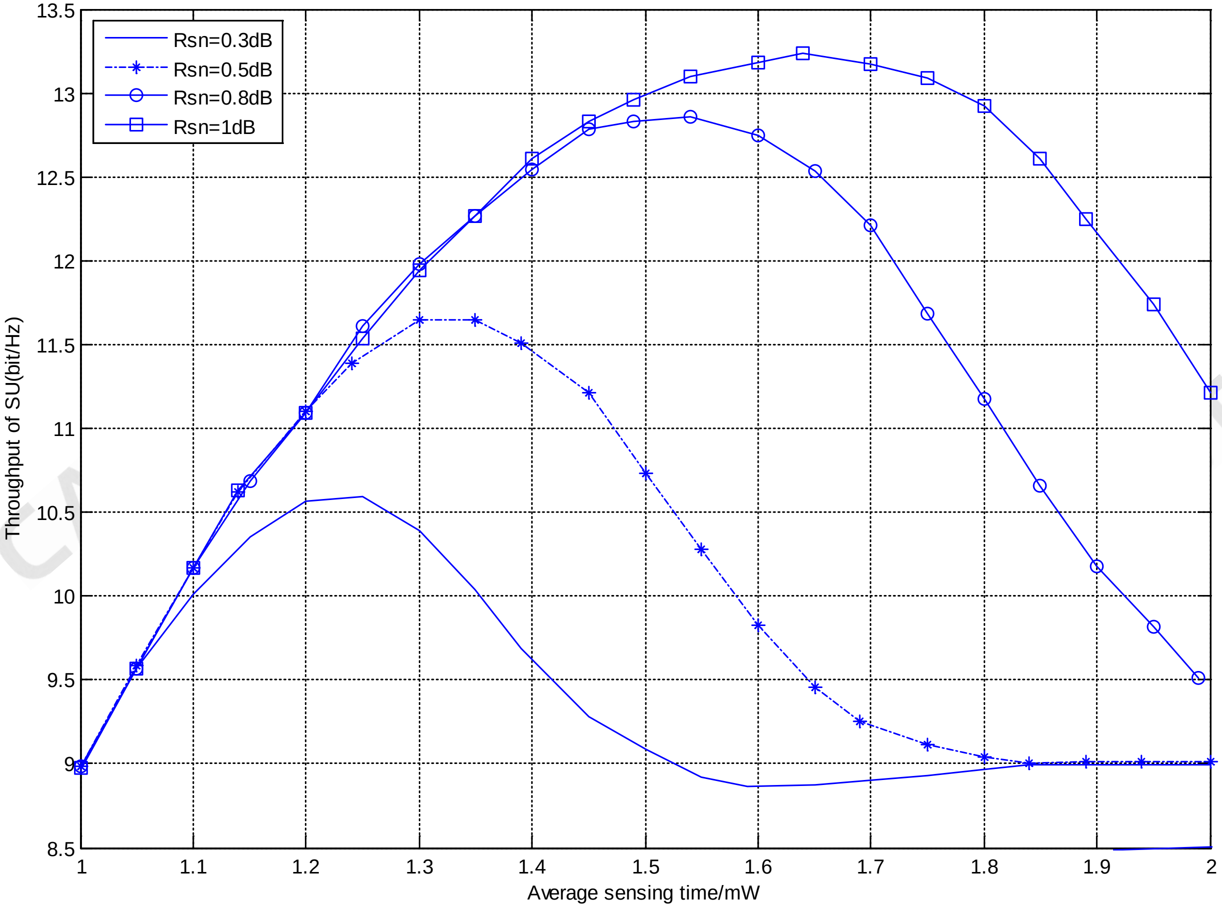
$$T = 1s, \sigma_l^2 = 1mW, L = 10, \alpha = 0.5, \beta = 0.9, P\left(H_l^0\right) = P\left(H_l^1\right) = 0.5$$

Transmit power of SU and PU is : 10mW

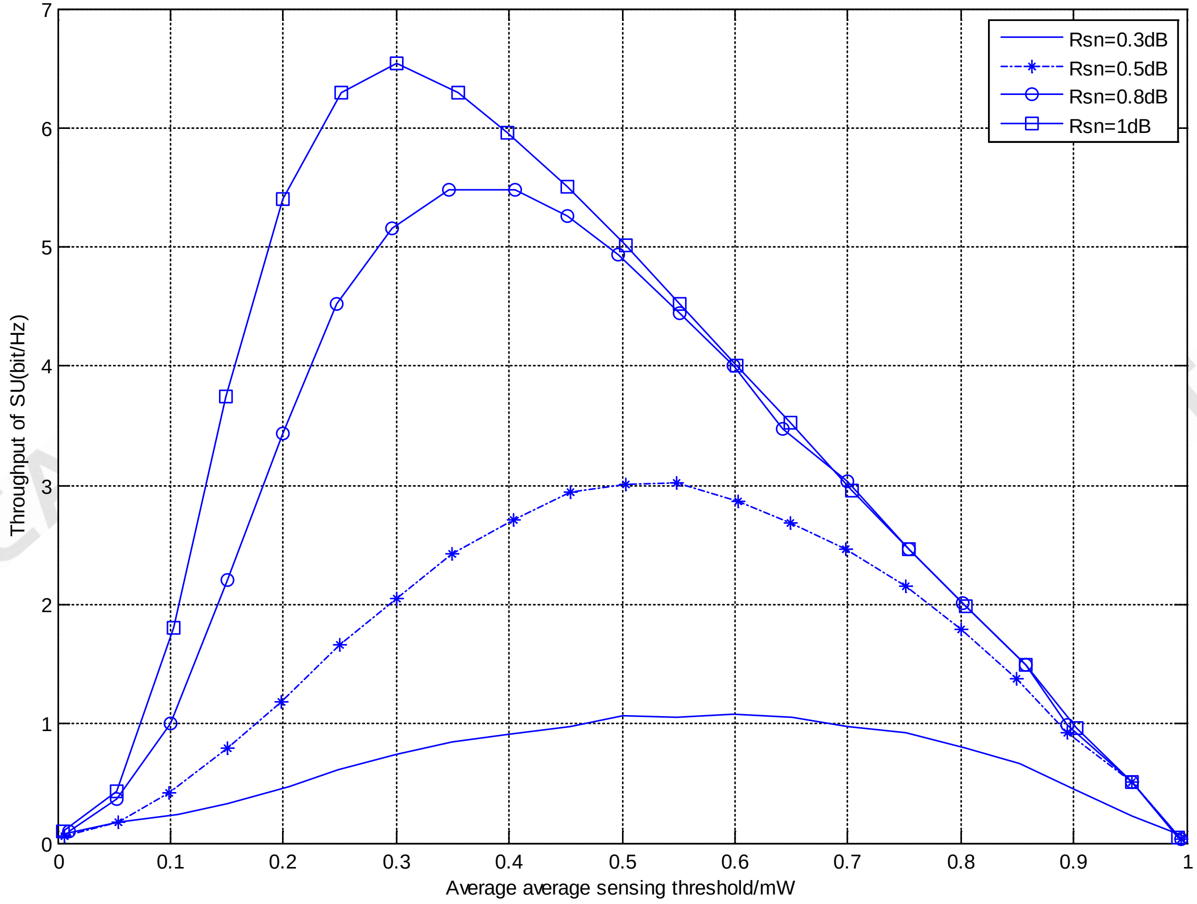
The multi-channel gain is Rayleigh distribution with mean -10dB

Estimation error: $\xi = 0.01$

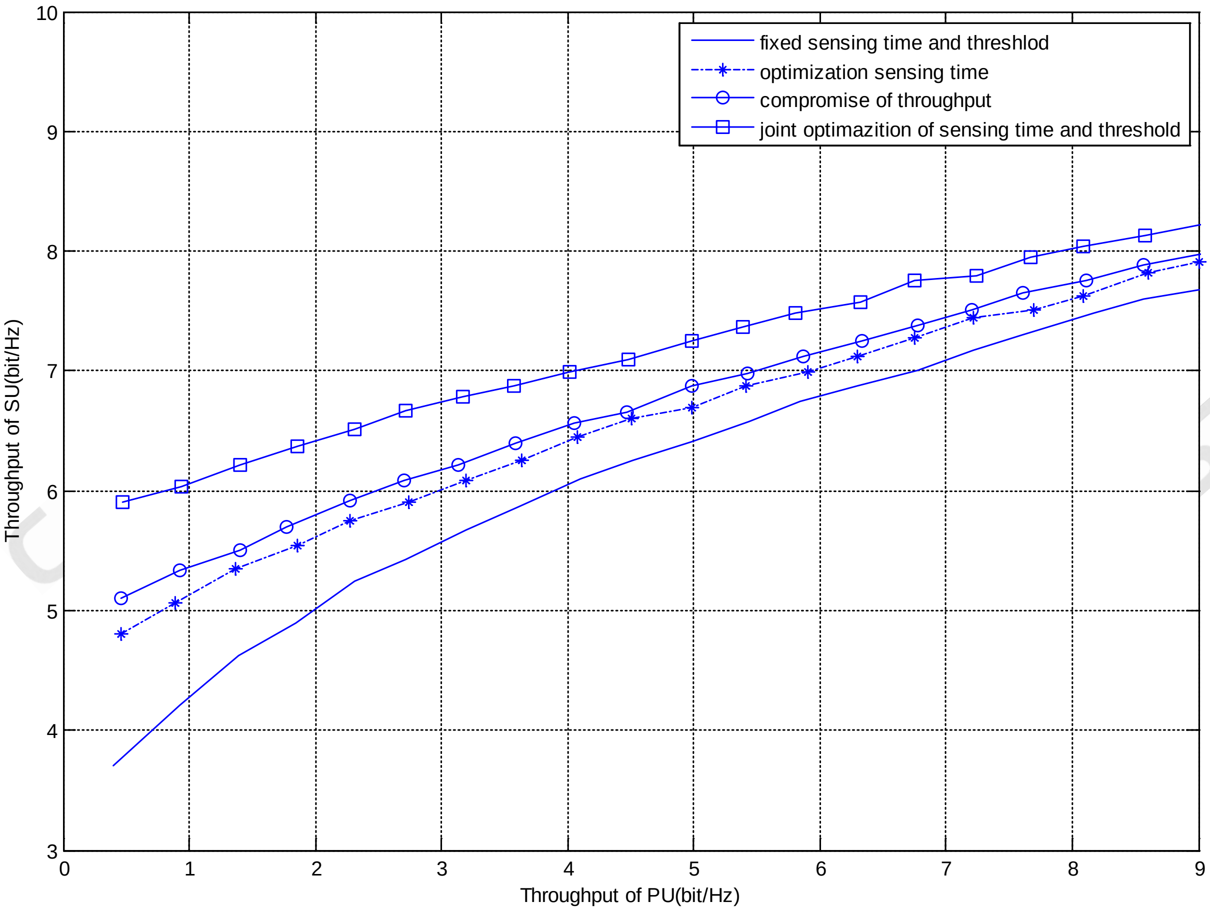
IV. Results And Discussion



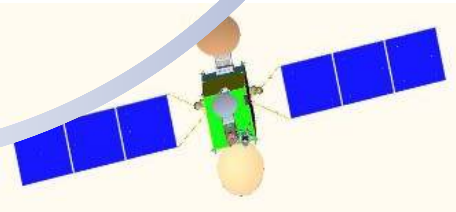
IV. Results And Discussion



IV. Results And Discussion



Conclusions



V. Conclusions

Based on the SU’s listen-before-transmit frame structure, a joint optimization of spectrum sensing time and threshold model to maximize the aggregate throughput of the SU over all the sub channels is proposed in this paper. The joint optimization algorithm alternatively optimizes sensing threshold and time to obtain the optimal solutions to the model.

The proposed method actually is optimization problems for sensing time and sensing threshold respectively. Theoretical analysis and simulation results show that these two optimization problems have a joint global optimal solution to maximize the throughput of SU. At a given throughput of PU, the proposed joint optimization method will obtain more throughput of SU.

"神舟"一号飞船升空
"Shenzhou-1" spacecraft lifting off

Thanks!